Modeling Thermal Stratification in Aquaculture Ponds

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Abstract

A stratification model was developed for the simulation of the temporal and spatial variation of temperature in aquaculture ponds. A new parameter, called the fish turbulent diffusivity, was incorporated in the equation to calculate the diffusion coefficient in this study. A simple, fast, yet robust numerical method was used to solve the equations of the model. The model, programmed in FORTRAN under the frame of the TRNSYS software, was implemented on a microcomputer. Simulation indicated that the fish turbulent diffusivity was important to the heat transfer estimation in aquaculture ponds. The model was tested using temperatures collected from two ponds in the suburbs of Beijing. The determination coefficient between the calculated and measured temperatures was about 0.90, while the standard deviation was about 0.2°C. The model is useful to better understand the evolution of thermal stratification and the process of heat energy and mass transfers in aquaculture ponds.

Introduction

Aquaculture ponds frequently undergo diel cycles of thermal and chemical vertical stratification and destratification under hot and quiescent weather conditions (Cathcart and Wheaton 1987; Boyd 1990; Szyper and Lin 1990; Losordo and Piedrahita 1991). Knowing the temperature profile in aquaculture ponds is important because water temperature directly affects the growth rate of all cultured species, and the development of density gradient in a water column aggravates the stratification of other water quality parameters, such as dissolved oxygen, pH, and alkalinity (Boyd 1990; Losordo and Piedrahita 1991). Thermal stratification thus plays a significant role in the system cycling of aquaculture ponds.
Previous attempts to characterize temperature patterns in aquaculture ponds were mostly performed using direct observations (Szyper and Lin 1990), or on the basis of the assumption of completely mixed pond conditions (Krant et al. 1982; Klemetson and Rogers 1985; Piedrahita 1990). Models with a fully uniform assumption over depth could not adequately describe temperature and water quality in stratified ponds.

There are, however, several reports on stratification models in aquaculture ponds (Cathcart and Wheaton 1987; Losordo and Piedrahita 1991; Culberson and Piedrahita 1996). The model of Cathcart and Wheaton (1987) was used to predict the vertical temperature profile of ponds. The turbulent mixing of heat downward through the water column was estimated using ‘wind-mixing’ algorithms, but the heat distribution through the water column was based only on the density gradient in the water column and not on wind surface shear calculations. In order to reduce the number of meteorological inputs, surface water temperature was used as a model input. In the model of Losordo and Piedrahita (1991), water column was divided into a number of horizontal layers (volume elements). Heat transfer by turbulent diffusion was modeled on the basis of a wind surface shear calculation. The fish movement influence on turbulent diffusion was not taken into account. Simulation results indicated that the model was less accurate in estimating the magnitude of thermal stratification (Losordo and Piedrahita 1991). The model of Culberson and Piedrahita (1996) was developed using a structure similar to that of Losordo and Piedrahita (1991), dealing with both temperature and dissolved oxygen patterns.

The purpose of this study was to develop a diel thermal stratification model for aquaculture ponds based on a finite layer structure of pond water and an hourly heat-energy balance of each layer by estimation of vertical turbulent diffusivity and heat transfer fluxes under various weather conditions. The model was different from the ones mentioned in the calculations of turbulent diffusivity and water surface heat transfers, and computer model implementation and verification.

Materials and Methods

Mathematical model

A finite water-layer structure was used to picture the spatial distribution of the dynamic variation of water temperature in aquaculture ponds. It was assumed that pond water was divided into several homogeneous layers of equal thickness, infinite in the horizontal plane. For each layer, a differential equation describing the time evolution of temperature was developed on the basis of energy balance, including the sink and source terms inside the layers as well as the transfer fluxes among the model layers and the boundary layers (Fig. 1). The water surface received solar radiation energy and exchanged sensible and latent heat with the air and thermal infrared radiation with the sky. The conductive heat transfer through the pond
bottom was neglected, and the volume and rate of water additions to ponds, necessary to maintain a constant water depth, was considered to be insignificant for heat balance. For an inner water layer \(1 < i < n\), the heat storage or dissipation flux density resulted from the difference between the incoming flux densities (the absorbed part of the solar radiation and the heat diffused from the upper layer) and the outgoing flux density (the heat diffused towards the lower layer) (Fig. 1). For the first layer, additional heat losses occurred by convection, evaporation (or condensation) and thermal radiation. Under our assumption of an adiabatic ground, the bottom layer did not undergo any heat loss. The heat transfer flux density between adjacent layers was supposed to be proportional to the temperature gradient and was depicted by a comprehensive diffusivity that included all the contributing transfer modes. Thus, the basic equations were derived as follows:

\[
C_w \rho \delta \frac{dT_i}{dt} = q_{SW1} - q_{W_a} - q_{LW,a} - q_{RW,sky} - q_{D1,2} \quad (1)
\]

\[
C_w \rho \delta \frac{dT_i}{dt} = q_{SWi} + q_{Di-1,i} - q_{Di,i+1} \quad (1 < i < n) \quad (2)
\]

\[
C_w \rho \delta \frac{dT_n}{dt} = q_{SWn} + q_{Dn-n} \quad (3)
\]

**Solar radiation**

The attenuation of solar radiation in the water column is commonly calculated using the Lambert-Beer law (Stefan et al. 1983; Cathcart and Wheaton 1987; and Losordo and Piedrahita 1991):

\[
q_{Swz} = S_{e} (1 - \rho_{Sw}) (1 - \beta) e^{-kz} \quad (4)
\]
Therefore, the solar radiation flux densities absorbed by each layer could be calculated below. The remaining solar radiation at pond bottom, usually very little for stratification ponds, was assumed to be entirely absorbed in the lowest water layer.

\[
q_{Sw} = \begin{cases} 
S_g (1 - \rho_{Sw}) (1 - \beta) (1 - e^{-k_i}) & (i = 1) \\
S_g (1 - \rho_{Sw}) (1 - \beta) (e^{-k_i} - e^{-k_i}) & (1 < i < n) \\
S_g (1 - \rho_{Sw}) (1 - \beta) e^{-k_i} & (i = n)
\end{cases}
\]

(5)

where \( \beta \) represents the fraction of solar radiation (the near infrared) absorbed near the water surface. Its value is typically about 0.4-0.5 (Losordo and Piedrahita 1991), and it was 0.45 in our model. Solar radiative reflectance at the water surface (\( \rho_{sw} \)) was calculated according to Ryan and Stolzenbach (1972) and Fritz et al. (1980):

\[
\rho_{Sw} = \begin{cases} 
1.18 \alpha^{-0.77} & (0 \leq F_{cn} \leq 0.1) \\
2.2 \alpha^{-0.97} & (0.1 < F_{cn} \leq 0.5) \\
0.95 \alpha^{-0.75} & (0.5 < F_{cn} \leq 0.9) \\
0.35 \alpha^{-1.45} & (0.9 < F_{cn} \leq 1)
\end{cases}
\]

(6)

where \( \alpha \) is the solar altitude angle (deg) computed by TRNSYS (Klein et al. 1990). \( F_{cn} \) is the cloudy factor of the sky, calculated by the ratio between diffuse and global solar radiation.

**Convection and evaporation**

The key issue for the calculation of convection and evaporation is to accurately estimate the transfer coefficients. They were considered to be directly proportional to wind speed in the models of Fritz et al. (1980), Losordo and Piedrahita (1991), and Culberson and Piedrahita (1996). In the present model, the convective heat flux density between the water surface and air was calculated according to the boundary layer theory:

\[
q_{Vw,a} = Nu \lambda \frac{T_j - T_a}{d}
\]

(7)

Latent heat exchange between water surface and air occurs with evaporation or condensation. The calculation of latent heat flux density (positive for evaporation and negative for condensation) was based on the same principle as convection:
The Nusselt (Nu) and Sherwood (Sh) numbers were calculated according to the convection mode (forced or natural) and flow type (laminar or turbulent) using the criteria proposed by Monteith and Unsworth (1990) (Table 1). Definitions and values of the dimensionless numbers for air at 20°C were given in Table 2. This method has been used to estimate convection and latent heat transfers of greenhouse systems by Nijskens et al. (1984), Pieters and Deltour (1997). It has been verified that the mean deviation between the calculated and measured air temperatures inside greenhouses was less than 1.0°C, with a standard deviation of less than 1.5°C (de Halleux et al. 1991).

Far infrared radiation

Although the sky is not a solid, it is usually treated as an equivalent black body (Nijskens et al. 1984). For a clear sky, the radiative sky temperature ($T_{sky}$) was modeled as (Swinbank 1963):

$$T_{sky} = 0.0552 (T_a + 273)^{1.5}$$

Table 1. Nusselt number for laminar and turbulent flow in natural and forced convection mode past a flat plate (Monteith and Unsworth 1990).

<table>
<thead>
<tr>
<th>Condition</th>
<th>Laminar flow</th>
<th>Turbulent flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Gr/Re^2 \geq 16$</td>
<td>$Gr \leq 10^5$</td>
<td>$Gr \leq 10^5$</td>
</tr>
<tr>
<td>Natural convection</td>
<td>$Nu = 0.54 (Gr Pr)^{1/4}$</td>
<td>$Nu = 0.15 (Gr Pr)^{1/3}$</td>
</tr>
<tr>
<td>$0.1 &lt; Gr/Re^2 &lt; 16$</td>
<td>$Sh = 0.54 (Gr Sc)^{1/4}$</td>
<td>$Sh = 0.15 (Gr Sc)^{1/3}$</td>
</tr>
<tr>
<td>Mixed convection</td>
<td>$Nu = 0.66Re^{1/2} Pr^{1/3}$</td>
<td>$Nu = 0.036Re^{4/5} Pr^{1/3}$</td>
</tr>
<tr>
<td>$Gr/Re^2 \leq 0.1$</td>
<td>$Sh = 0.66Re^{1/2} Sc^{1/3}$</td>
<td>$Sh = 0.036Re^{4/5} Sc^{1/3}$</td>
</tr>
</tbody>
</table>

Table 2. Definitions and values of some dimensionless numbers for air at 20°C.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Definition</th>
<th>Value for air at 20°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grashof</td>
<td>$Gr$</td>
<td>$g B_{air} d^3 (T_i - T_a) \nu^{-2}$</td>
<td>1.576 x 10^8 d^3 (T_i - T_a)</td>
</tr>
<tr>
<td>Prandtl</td>
<td>$Pr$</td>
<td>$v/D_a$</td>
<td>0.7020</td>
</tr>
<tr>
<td>Reynolds</td>
<td>$Re$</td>
<td>$W_d/v$</td>
<td>6.623 x 10^4 W d</td>
</tr>
<tr>
<td>Schmidt</td>
<td>$Sc$</td>
<td>$v/D$</td>
<td>0.63</td>
</tr>
</tbody>
</table>
For a total overcast sky, $T_{sky}$ was supposed to be equal to the air temperature (Nijskens et al. 1984). For a partially cloudy sky, $T_{sky}$ was given as a weighted-mean value by the cloudy factor of the sky:

$$T_{sky} = (T_a + 273) F_{cn} + 0.0552 \ (T_a + 273)^{1.5} (1 - F_{cn})$$  \hspace{1cm} (10)$$

Therefore, according to the Stefan-Boltzmann law, the thermal radiative flux density between the water surface and the sky could be expressed as:

$$q_{Rw,sky} = \varepsilon_w \sigma (T_1 + 273)^4 - \sigma T_{sky}^4$$  \hspace{1cm} (11)$$

where $\varepsilon_w$ is emissivity of water surface, $\varepsilon_w = 0.97$ according to Fritz et al. (1980), Losordo and Piedrahita (1991). Simulation results showed that the presence of cloud cover was a major factor in affecting the thermal radiative transfer between the water surface and the sky.

**Heat diffusion**

A well-known Fickian diffusion equation was used to calculate the heat diffusion transfer between two adjacent water layers:

$$q_{D,i+1} = C_w \rho_w \frac{D_{i,i+1}}{\delta} (T_i - T_{i+1})$$  \hspace{1cm} (12)$$

where $D_{i,i+1}$ is comprehensive diffusivity, including molecular and turbulent diffusion. The estimation accuracy of this coefficient is considered to be of primary importance for the success of modeling the stratified ponds and the diffusion transfers of heat and mass. A parameter, called the effective diffusion coefficient ($E_Z$), was used in the model of Losordo and Piedrahita (1991), and Culberson and Piedrahita (1996). It was modeled as a function of wind speed, pond depth and temperature gradient (Sundaram and Rehm 1973):

$$E_Z = E_{0,Z} (1 + 0.1 \frac{Ri_Z}{Z})^{-1}$$  \hspace{1cm} (13)$$

where $E_{0,Z}$ is the effective diffusion coefficient at neutral stability at a depth of $Z$. $Ri_Z$ is the Richardson number at a depth of $Z$. Further descriptions of these two parameters can be found in Henderson-Sellers (1984) and Sundaram and Rehm (1973).

Simulation results indicated that the above model could not accurately estimate the magnitude of thermal stratification. The profile of simulated temperature was generally much more stratified than the measured data (Fig. 2). A similar result has been reported by Losordo and Piedrahita (1991). In fact, the impact of fish movement on turbulent diffusion was not
taken into account in Eq. (13). Therefore, a new parameter, called the fish turbulent diffusivity ($D_{f}$), was incorporated in the present model. The total comprehensive diffusivity at a depth of $Z$ was modeled as:

$$D_Z = \sqrt{D_i^2 + E_Z^2}$$

(14)

For the diffusion of two adjacent layers, the diffusivity ($D_{ii+1}$) was calculated using Eq. (14) at the depth of $Z = i \delta$ (see Fig. 1). Here $i =$ the number of water layers, and $\delta =$ the thickness of each water layer.

The accurate estimation of fish turbulent diffusivity is crucial to the success of the stratification model. The parameter may vary with fish characteristics within a pond. If there was no significant difference of fish biomass, however, the value would be estimated by minimizing the quadratic error criterion between the model output ($T_{C_{ij}}$) and the temperature measurements ($T_{m_{ij}}$) of the water layers in a stratified pond:

$$Q = \sum_{j=1}^{m} \sum_{i=1}^{n} (T_{C_{ij}} - T_{m_{ij}})^2$$

(15)

**Model implementation**

The above differential equations are linked together by energy or mass fluxes. The method of solution is crucial to the modeling process. The Runge-Kutta method was used by Fritz (1985) and Piedrahita (1990). In this study, a different method was used, which provided for faster computation and simplicity in programming. Each of the equations was simplified as a first-order differential equation with constant coefficients over a time step. If the flux terms of any variable equation included the other variables, their values were then computed using the variable values from the previous time step.

Fig. 2. When using the diffusion coefficient estimated by the equation of Sundaram and Rehm (1973), simulated temperature profile was much more stratified than the measurements of pond A in the suburbs of Beijing.
Thus, the explicit solutions of the simplified equations could be easily derived. For example, the temperature equation of an inner layer (1< i< n) can be simplified as follows:

\[
\frac{dT_i}{dt} = aT_i + b
\]  \hspace{1cm} (16)

with:

\[
a = -(D_{i,i} + D_{i+1,i})/\delta^2
\]  \hspace{1cm} (17)

\[
b = -\frac{q_{swi}}{C_up} + \frac{1}{\delta^2} [D_{i-1,i}(t-\Delta t)T_{i-1}(t-\Delta t) + D_{i,i+1}(t-\Delta t)T_{i+1}(t-\Delta t)]
\]  \hspace{1cm} (18)

where \(T_{i}(t-\Delta t)\) and \(D_{i,i+1}(t-\Delta t)\) are the relative values of \(T_i\) and \(D_{i,i+1}\) at previous time step. For the above equation, \(a\) and \(b\) are constants over a time step, and the explicit solution was thus obtained as:

\[
T_i = \begin{cases} 
T_{i}(t-\Delta t) + b\Delta t & (a = 0) \\
\frac{b}{a}[T_{i}(t-\Delta t) + b/a]\exp(a\Delta t) - b/a & (a \neq 0)
\end{cases}
\]  \hspace{1cm} (19)

The computer program of the model, written in FORTRAN language, stood in the frame of the TRNSYS modular transient simulation program (Klein et al. 1990), which is used for the simulation and analysis of the time-dependent phenomena, especially in the domain of heat and mass transfers. Execution of the model showed that the convergence could be stable when the time step was less than 0.05 h.

During the day, stable water temperature stratification results from the progressive extinction of the solar radiation with depth. This continuous procedure was pictured by the step evolution of the layer characteristics. During the night, water stratification is destroyed with the inversion of the vertical gradient of water density caused by water surface cooling. The pond water, therefore, gradually mixes from top to bottom. This can also happen during the day, if there is rain or heavy wind. Therefore, during each time step of simulation, temperatures of all water layers were examined, and if \(T_1 < T_2\), then each took their mean value. This process was continued within the time step until \(T_1 = T_2 = \ldots = T_{i+1}\). Thus, the water from layer 1 to layer \(i\) was mixed, but stratification still existed below layer \(i+1\). If the convective mixing reached the last layer, then the pond water was wholly homogeneous.

**Model verification**

Data were collected in ponds A and B at a fish farm in the suburbs of Beijing from 25 to 30 in July 1994 (Table 3 and Fig. 3). The global solar radiation was automatically recorded using a LI-1776 solar monitor (Li Cor.,
USA) by taking integral values of 0.5 h. The extinction coefficient of underwater solar radiation was calculated using linear regression of logarithmic values of underwater PAR (Photosynthetically Active Radiation) which was from an underwater PAR sensor (Li Cor., USA). Dry and wet bulb air temperatures were obtained using an Assmman psychrometer. Wind speed was measured at 2 m above the water surface using a QDF-3 hot-wire anemometer. Water temperature was collected from 0.1 m depth to the bottom at 0.2 m intervals every 2 hours on consecutive diurnal cycles using the temperature sensor of a YSI-58 DO meter (Yellow spring Inc., USA).

Results and Discussion

Simulation was carried out using a set of data inputs (Table 4 and Fig. 3). To match the corresponding measured temperatures, water in pond A was divided into ten layers. In order to obtain optimal fish turbulent diffusivity ($D_f$), the minimum value of Eq. (15) was searched step-by-step using different values of $D_f$ with a small increment step of $5 \times 10^{-7} \text{m}^2\text{s}^{-1}$ (Thermal diffusivity in steady state water at 20°C is about $1.4 \times 10^{-7} \text{m}^2\text{s}^{-1}$ (Kays and Crawford 1980)). An optimal fish turbulent diffusivity was thus found for pond A, $D_f = 1.9 \times 10^{-5} \text{m}^2\text{s}^{-1}$, using two hundred forty pieces of stratification temperatures gathered over four days.

Table 3. Pond conditions of ponds A and B in the suburbs of Beijing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pond A</th>
<th>Pond B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude (°N), Longitude (°E)</td>
<td>40, 116</td>
<td>40, 116</td>
</tr>
<tr>
<td>Pond size (m$^2$)</td>
<td>6000</td>
<td>3300</td>
</tr>
<tr>
<td>Water depth (m)</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Mean fish size (g fish$^{-1}$)</td>
<td>325</td>
<td>312</td>
</tr>
<tr>
<td>Fish biomass density (g m$^{-3}$)</td>
<td>338</td>
<td>365</td>
</tr>
<tr>
<td>Common carp (Cyprinus carpio)</td>
<td></td>
<td>(265)</td>
</tr>
<tr>
<td>Grass carp (Ctenopharyngodon idella)</td>
<td>(201)</td>
<td></td>
</tr>
<tr>
<td>Silver carp (Hypophthalmichthys molitrix) and bighead (Aristichthys nobilis)</td>
<td>(137)</td>
<td>(100)</td>
</tr>
</tbody>
</table>

Fig. 3 Site climatic conditions of the fish farm in the suburbs of Beijing (40°N, 116°E) 1994.
The accuracy of the model was evaluated through a comparison between simulated and measured temperatures. The parameters used to assess the accuracy of simulation included the determination coefficient of linear regression ($r^2$) and standard deviation.

The temporal and vertical temperature variations of pond A were graphed in Fig. 4. Measured temperatures were abnormally low in the afternoon of 29 July due to an accidental cold water (14.9°C) input from a nearby well. Data collected during that period were thus excluded from the statistical calculation. For the total data samples of the ten layers (N=480), the determination coefficient was $r^2 = 0.93$ (Fig 5). The maximum value of $r^2$ was 0.95 at layer 4 (N = 48), while the minimum was 0.9 at layer 9. Among all the water layers, the standard deviation was from 0.10°C (layer 5) to 0.16°C (layer 1). The maximum value of standard deviation appeared at the surface layer (Fig. 5), because the first layer had the largest temperature fluctuation compared with the other layers.

If we compared Fig. 4 with Fig. 2, it would be clear that the accuracy of the model was vastly improved due to the introduction of the fish turbulent diffusivity. Fish movement was thus an important role in the heat transfer estimation for a stratification aquaculture pond.

Table 4. Data inputs for simulation of ponds A and B in the suburbs of Beijing.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Pond A</th>
<th>Pond B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude (°N)</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Climatic conditions (Fig. 3)</td>
<td>(Fig. 3)</td>
<td>(Fig. 3)</td>
</tr>
<tr>
<td>Pond water depth (m)</td>
<td>2.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Solar radiation extinction coefficient (m$^{-1}$)</td>
<td>5.94</td>
<td>4.74</td>
</tr>
<tr>
<td>Fish turbulent diffusivity (m$^2$ s$^{-1}$)</td>
<td>$1.9 \times 10^{-5}$</td>
<td>$1.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>Water layer thickness (m)</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Julian day</td>
<td>206</td>
<td>206</td>
</tr>
<tr>
<td>Initial time (h)</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Initial temperature of each water layer (°C)</td>
<td>30.8</td>
<td>30.5</td>
</tr>
</tbody>
</table>

Fig. 4. Simulated and measured temperature profiles of pond A in the suburbs of Beijing. Due to an accidental cold water (14.9°C) input from a nearby well in the afternoon of 19 July, measured temperatures were abnormally lower than simulated data, especially at the deeper layers.
For verification of the model accuracy, simulation was also carried out for pond B. The same value of fish turbulent diffusivity \( (D_f = 1.9 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}) \) was used for the simulation of pond B, since the difference of fish biomass densities between ponds A and B was less than 8% (Table 3). The results were plotted in Figs. 6 and 7. The determination coefficient was \( r^2 = 0.91 \) for the total data samples \( (N = 648) \). The value of \( r^2 \) was around 0.9 for the upper eight layers, but gradually decreased with depth for the lower layers, while the minimum value was 0.58 at layer
The total standard deviation was 0.18°C (Fig. 7), a little bit larger than that of pond A (Fig. 5). The standard deviation ranged from 0.12°C (layer 7) to 0.26°C (layer 1). The maximum values of the two parameters still occurred at the first layer.

**Nomenclature**

- $C_{Lw}$: latent heat of water evaporation (J kg$^{-1}$)
- $C_w$: mass thermal capacity of water (J kg$^{-1}$ °C$^{-1}$)
- $d$: characteristic length (m)
- $D$: molecular diffusion coefficient of water vapour in air (m$^2$ s$^{-1}$)
- $D_a$: thermal diffusivity of air (m$^2$ s$^{-1}$)
- $D_f$: fish turbulent diffusivity (m$^2$ s$^{-1}$)
- $D_{i, i+1}$: comprehensive diffusivity between layers $i$ and $i+1$ (m$^2$ s$^{-1}$)
- $D_{i, i+1}(t-\Delta t)$: comprehensive diffusivity at previous time step (m$^2$ s$^{-1}$)
- $D_Z$: comprehensive diffusivity at depth of $Z$ (m$^2$ s$^{-1}$)
- $e_a$: water content of air (kg m$^{-3}$)
- $e^{*}_{w1}$: saturated water content of air at temperature of layer 1 (kg m$^{-3}$)
- $E_Z$: effective diffusion coefficient at depth of $Z$ (m$^2$ s$^{-1}$)
- $E_{0, Z}$: effective diffusion coefficient at neutral stability at depth of $Z$ (m$^2$ s$^{-1}$)
- $F_{cn}$: cloudy factor of the sky (dimensionless)
- $g$: gravitational acceleration (m s$^{-2}$)
- $Gr$: Grashof number (dimensionless)
- $k$: extinction coefficient of solar radiation (m$^{-1}$)
- $N$: sample number
- $Nu$: Nusselt number (dimensionless)
- $Pr$: Prandtl number (dimensionless)
- $Q$: quadratic error
- $q_{Di, i+1}$: diffusion heat flux density between layers $i$ and $i+1$ (W m$^{-2}$)
- $q_{Lw,a}$: latent flux density between water surface and air (W m$^{-2}$)
- $q_{Rw,sky}$: thermal radiative flux density between water surface and sky (W m$^{-2}$)
- $q_{Swz}$: solar radiation flux density at depth of $Z$ (W m$^{-2}$)
- $q_{Swi}$: solar radiation flux density absorbed by layer $i$ (W m$^{-2}$)
- $q_{Vw,a}$: convective flux density between water surface and air (W m$^{-2}$)
- $Re$: Reynolds number (dimensionless)
- $Ri_Z$: Richardson number at depth of $Z$ (dimensionless)
- $Sc$: Schmidt number (dimensionless)
- $S_g$: global solar radiation (W m$^{-2}$)
- $Sh$: Sherwood number (dimensionless)
- $t$: time (s)
- $T_a$: air temperature (°C)
- $T_i$: temperature of layer $i$ (°C)
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\( T_i(t-\Delta t) \)  temperature of layer i at previous time step (°C)

\( T_{sky} \)  radiative sky temperature (K)

\( W \)  wind speed (m s\(^{-1}\))

\( Z \)  water depth (m)

\( a \)  solar altitude angle (deg)

\( b_{air} \)  expansion coefficient of air (K\(^{-1}\))

\( b \)  fraction of solar radiation (near infrared) absorbed near water surface (dimensionless)

\( d \)  thickness of each water layer (m)

\( e_w \)  emissivity of water surface (dimensionless)

\( l \)  thermal conductivity of air (W m\(^{-1}\) °C\(^{-1}\))

\( n \)  kinematic viscosity of air (m\(^2\) s\(^{-1}\))

\( r_{Sw} \)  solar radiative reflectance of water surface (dimensionless)

\( r_w \)  water density (kg m\(^{-3}\))

\( s \)  Stefan-Boltzmann constant (5.67 \times 10^{-8} \text{ W m}^2 \text{ K}^{-4})

\( Dt \)  time step of simulation (s)

**Conclusions**

1. The stratification model developed in this study seemed reasonable to better understand the temporal and spatial evolution of temperature in aquaculture ponds. Model tests showed that the determination coefficient between the computed and measured temperatures was about 0.90, while standard deviation was about 0.2°C.

2. A new parameter, the fish turbulent diffusivity, was incorporated in the equation to calculate the comprehensive diffusivity. It was determined by minimization of the quadratic error between calculated and measured temperatures in this study. Simulation indicated that fish turbulent diffusivity was crucial to the heat transfer estimation in a stratified pond.

3. Hourly accurate estimations of heat transfers were important in modeling a stratified pond. Heat fluxes were modeled hourly in this study, including convection and latent heat transfers between water surface and air, and thermal radiation between water surface and sky.

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